Wiffleball size

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Abstract

This is a little attempt at calculating the size of the wiffleball at $\beta = 1$ by Indrek. This is based on a load of assumptions so should be considered as an estimate. But it does give us good/sane numbers.

1 Image coil current

With notation from [1] we have factor

$$\xi = \frac{a^2}{r_p^2 + \left(r_p + \frac{s_p}{\sqrt{2}}\right)^2} = \frac{a^2}{c_p^2}$$

where

$$c_p^2 = r_p^2 + \left(r_p + \frac{s_p}{\sqrt{2}}\right)^2$$

and the image coil current is

$$I_i = -\frac{I_p}{\sqrt{\xi}} = -\frac{I_p c_p}{a}.$$

The image coil radius is

$$r_i = r_p \xi = \frac{r_p a^2}{c_p^2}$$

2 Number of electrons needed for the image coils

For/from the dicussion at [2] the electron number calculation based on image coil theory would be following.

We start with these assumptions from [3]:
A=0.150000
R=0.150000, S=0.080000, I=200000.000000
iR=0.051787, iS=0.027620, iI=-340380.343468
You can derive your own using formulae in [1].

Image coil current:

$$|I_i| = 340000 = 3.4e5\ A.$$

Image coil radius:

$$r = \frac{r_p a^2}{c_p^2} = 0.051787\ m.$$

Image coil circumference:

$$C = 2\pi r = 0.325\ m.$$

Elementary/electron charge:

$$e_- = 1.6e\text{-}19\ C$$

Energy of an electron in eV:

$$E_V = 12keV = 1.2e4\ eV$$

Energy of an electron:

$$E = E_V \cdot e_- = 1.2e4 \cdot 1.6e\text{-}19\ J \approx 1.92e\text{-}15\ J$$

Speed of electrons:

$$E = \frac{m_e v^2}{2} \Rightarrow v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2E_v e_-}{m_e}} = \sqrt{\frac{2 \cdot 1.2e4 \cdot 1.6e\text{-}19}{9.12e\text{-}31}} \approx 6.5e7\ m/s.$$

Current from one electron inside one image coil:

$$I_1 = \frac{v}{C} \cdot e_- = \sqrt{\frac{2\kappa E_v e_-}{m_e}} \cdot \frac{e_-}{C} \approx 6.5e7 \cdot \frac{1.6e\text{-}19}{0.325} \approx 3.2e\text{-}11\ A.$$

Number of electrons required to provide full coil current:

$$N_c = \frac{|I_i|}{I_1} = \frac{|I_i|\,C}{e}\sqrt{\frac{m_e}{2E}} = \frac{I_p c_p}{a} 2\pi r \sqrt{\frac{m_e}{2E_v e_-^3}} \approx 1.06e16.$$

Total amount of electrons:

$$N = 6 \cdot N_c = \frac{12 I_p c_p \pi r}{a}\sqrt{\frac{m_e}{2E_v e_-^3}} = \frac{12 I_p \pi r_p a}{c_p}\sqrt{\frac{m_e}{2E_v e_-^3}} \approx 6.4e16.$$

This is the ideal lower bound to the number of electrons.

3 Magnetic pressure

The force on a physical coil by all the other physical coils is a line integral over the coil against the magnetic field of all the other physical coils:

$$\mathbf{F}_p = \int_{coil} I d\mathbf{l} \times \mathbf{B}_p.$$

The force on a physical coil by all the otehr physical coils and a wiffleball is a line integral over the coil against the magnetic field of the other physical and the image coils:

$$\mathbf{F}_{p+i} = \int_{coil} I d\mathbf{l} \times \mathbf{B}_{p+i}.$$

The net force pushing outwards between wiffleball and physical-only coils is

$$F_{net} = 6 \cdot |\mathbf{F}_{p+i} - \mathbf{F}_p|.$$

The magnetic pressure is

$$P = \frac{F_{net}}{S} = \frac{6 \cdot |\mathbf{F}_{p+1} - \mathbf{F}_p|}{4\pi a^2}.$$

The idea to do this like this is from icarus, see [5].

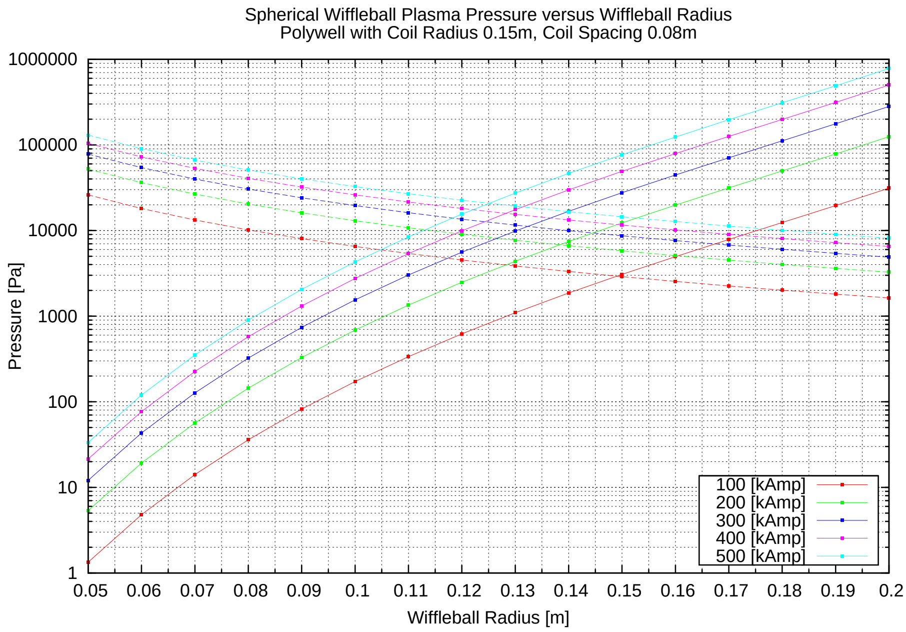
4 Electron kinetic pressure

Pressure from the electrons. We get the pressure equation from the kinetic theory ([4]):

$$\begin{aligned} P &= \frac{nm_e\overline{v^2}}{3} = \frac{N}{V}\frac{m_e}{3}\overline{v^2} = \\ &= N\frac{3}{4\pi a^3}\frac{m_e}{3}\frac{2E_v e_-}{m_e} = \frac{12 I_p \pi r_p a}{c_p}\sqrt{\frac{m_e}{2E_v e_-^3}}\frac{1}{4\pi a^3}2E_v e_- = \\ &= \frac{6 I_p r_p}{c_p}\sqrt{\frac{m_e E_v}{2e_-}}\frac{1}{a^2}. \end{aligned}$$

Now I've neglected the pressure from ions. Perhaps one should consider them too. I think this would at worst case double the pressure (assuming same number of ions as electrons). But the mantra has been that ions at the edge are slow, so won't affect the pressure that much (if you can believe that). However, even doubling the pressure won't significantly change the wiffleball size (it equates to doubling the current - see the graph below).

5 Magnetic pressure, electron kinetic pressure, beta=1



This is for WB6 dimensions and properties. Here the solid line is magnetic pressure, dashed line is the electron kinetic pressure. Where lines of same color cross we have $\beta = 1$. This is with $E_V = 12kV$.

References

[1] http://www.mare.ee/indrek/ephi/images.pdf

[2] http://www.talk-polywell.org/bb/viewtopic.php?t=2086

[3] http://www.mare.ee/indrek/ephi/invwb/

[4] http://en.wikipedia.org/wiki/Kinetic_theory#Pressure

[5] http://www.mare.ee/indrek/ephi/force2/, http://www.mare.ee/indrek/ephi/invwb/